



## 50 Important Math Concepts You Must Know for the ACT

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*The ACT Mathematics exam provides a comprehensive “tour” through the foundation courses in secondary school Mathematics. From pre-Algebra, through Geometry, and the concepts of Algebra II and Trigonometry, virtually everything a student has seen in high school math will be tested. This exam is 60 minutes long and has 60 questions. Each question has five choices. You may use a calculator, preferably a graphing one. You may not use the TI-89 or the TI-nspire CAS.*

### ARITHMETIC AND ALGEBRA I (about 40% of the exam)

- 1. Prime Numbers** are numbers that are only divisible by themselves and one:
  - 1 **IS NOT** a prime number
  - 2 is the **only** even prime number
- 2. Even/Odd Rules:**
  - The product of two even numbers is always even
  - The product of two odd numbers is always odd
  - The sum or difference of two even numbers is always even
  - The sum or difference of two odd numbers is always even
- 3. Consecutive Integers:**
  - $x, x+1, x+2$ , etc. represent consecutive integers
  - $x, x+2, x+4$ , etc. represent **even or odd** consecutive integers
- 4. Mean (Average):**
  - The sum of the values divided by the number of values
  - The average of two groups of data values is NOT the average of the two averages
- 5. Median:**
  - The middle value of a group of values, when ordered increasingly
  - If you have an even number of values, then take the average of the middle two terms
- 6. Mode:**
  - The most commonly occurring value in a group
  - A set of values can have more than one mode
- 7. Remainder:**
  - The integer quantity left over when dividing a quantity by a divisor
  - A remainder is never a decimal, and is never bigger than the divisor

**8. Range:**

- The difference between the highest and lowest values in a data set

**9. Percent Change:**

- (amount of change) / (initial value) \* 100%
- Remember that percent values never add or subtract

**10. Rational Numbers:**

- Any decimal, repeating or non-repeating, that can be written as the quotient of two integers

**11. Irrational Numbers:**

- Any non-repeating decimal that does not have a predictable pattern to it

**12. Linear Inequalities:**

- When solving these, if you divide or multiply by a negative value, you must reverse the sign of the inequality

**13. Rules of Exponents:**

- Bases must be alike
- For any base  $x$ ,  $x^0 = 1$
- When multiplying exponents, add the bases:  $x^a * x^b = x^{a+b}$
- When dividing exponents, subtract the bases:  $x^a / x^b = x^{a-b}$
- When taking a power of exponents, multiply them:  $(x^a)^b = x^{ab}$
- A square root is the same thing as a  $\frac{1}{2}$  power:  $25^{1/2} = 5$
- A cube root is the same thing as a  $\frac{1}{3}$  power:  $125^{1/3} = 5$

**14. Factoring Simple Polynomials in the form  $x^2 + bx + c$ :**

- The two factors must multiply to  $c$  and add to  $b$

**15. Factoring Factors in the form  $x^2 - y^2$ :**

- The two factors are always  $(x + y)(x - y)$

**PLANE AND COORDINATE GEOMETRY (about 40% of the exam)****16. Triangle Rules:**

- The angles of a triangle always add to 180 degrees
- The sum of any two sides of a triangle must exceed the measure of the third side
- In a scalene triangle, all sides and angles are different:
  - The larger the side, the larger the angle opposite that side
- Properties of isosceles triangles:
  - The base angles are congruent (equal in measure)
  - Sides opposite base angles are congruent
  - The angle bisector drawn from the vertex angle forms a perpendicular bisector

**17. Properties of right triangles:**

- Pythagorean Theorem:  $a^2 + b^2 = c^2$
- Important Pythagorean triples to know:
  - 3, 4, 5
  - 5, 12, 13
  - 8, 15, 17
  - 7, 24, 25
  - 9, 40, 41
  - **Multiples of Pythagorean Triples are also Pythagorean Triples!**

**18. Triangle Exterior Angle Theorem:**

- The exterior angle of a triangle is equal to the sum of the two opposite interior angles.

**19. Classifying Triangles by Side Lengths:**

- Order the sides from least to greatest: a, b, c, respectively
- Compare the sum of  $a^2 + b^2$  to  $c^2$ :
- If  $a^2 + b^2 > c^2$ , then the triangle is **acute**
- If  $a^2 + b^2 = c^2$ , then the triangle is **right**
- If  $a^2 + b^2 < c^2$ , then the triangle is **obtuse**

**20. Parallelogram Attributes:**

- A parallelogram is a quadrilateral in which opposite sides are congruent and parallel
- Its adjacent angles add up to 180 degrees and opposite angles are congruent
- Its diagonals bisect each other

**21. Special Parallelograms:**

- Rectangle
  - All angles are right angles and diagonals are congruent
- Rhombus
  - All sides are congruent and diagonals are perpendicular
- Square
  - All sides are congruent; diagonals are congruent and perpendicular
  - All angles are right angles

**22. Lines and angles:**

- A straight line has 180 degrees
- Two supplementary angles on a straight line add to 180 degrees
- A right angle has 90 degrees
- Two complementary angles on a right angle add to 90 degrees
- Vertical angles are angles formed by intersecting lines and are congruent
- Angles that meet at a point all add up to 360 degrees

**23. Parallel Lines:**

- Parallel lines never meet
- Two parallel lines are cut by a **transversal**:
  - Corresponding angles form an F and are congruent
  - Alternate interior angles form a Z or N and are congruent
  - Same side interior / exterior angles are supplementary

**24. Circles:**

- A circle has 360 degrees
- The radius of a circle extends from the center to the edge
- The diameter is twice the length of the radius
- The **circumference** of a circle is  $2\pi r$ 
  - **Arc length:**  $(n/360) * 2\pi r$ ; n = number of degrees in the arc
- The **area** of a circle is  $\pi r^2$ :
  - **Sector Area:**  $(n/360) * \pi r^2$ ; n = number of degrees in the central angle
- When you double a radius:
  - You double its circumference
  - You **quadruple** its area
- Frequently tested angles on the ACT:
  - **Central Angle**
    - Formed by two radii
    - Equal to the number of degrees in the intercepted arc
  - **Inscribed Angle**
    - Formed by two chords at the edge of a circle
    - Equal to half of the number of degrees in the intercepted arc
  - **Angle Inscribed in a semicircle**
    - Always  $90^\circ$
  - **Angle formed by a radius and tangent line to a circle**
    - Always  $90^\circ$

**25. Important Area Formulas:**

- Triangle:  $\frac{1}{2}bh$
- Rectangle:  $bh$
- Trapezoid:  $\frac{1}{2}h(b_1 + b_2)$  [sum of the bases]
- Rhombus:  $\frac{1}{2}d_1d_2$  (product of the diagonals)

**26. Important Volume Formulas:**

- Cube:  $s^3$
- Box:  $lwh$
- Cylinder:  $\pi r^2h$
- Sphere:  $\frac{4}{3}\pi r^3$
- Cone:  $\frac{1}{3}\pi r^2h$

**27. Important Surface Area Formulas:**

- Box:  $2lw + 2lh + 2wh$ 
  - $d^2 = l^2 + w^2 + h^2$  (length of a diagonal of a box)
- Cube:  $6s^2$

**28. When a circle is inscribed in a square the diameter of the circle is the same as the length of one side of the square**

**29. When a square is inscribed in a circle, the diagonal of the square is the same as the diameter of the circle.**

**30. Important Measurements in Polygons:**

- The number of degrees in any n-gon is  $180(n - 2)$ :
  - A triangle has 180 degrees, a quadrilateral has 360 degrees, a pentagon has 540 degrees, etc.
  - In a regular n-sided polygon (where all the sides and angles are equal), the measure of any angle is given as  $\frac{360}{n}$
  - The number of diagonals in any n-sided polygon is  $\frac{1}{2}n(n - 3)$ . This is a little known but frequently tested concept!

**31. Isosceles Right Triangles:**

- Base angles each measure 45 degrees
- If you are given the leg, the hypotenuse is the leg  $\sqrt{2}$
- If you are given the hypotenuse, the leg is the hypotenuse/ $\sqrt{2}$

**32. 30/60/90 right triangles:**

- The ratio of the sides of a 30/60/90 triangle is  $1: \sqrt{3} : 2$

**33. Similar Shapes**

- All corresponding angles are congruent
- Sides and perimeters are in proportion. This is called the **ratio of similarity**
- The areas are in proportion, too and is the **(ratio of similarity)<sup>2</sup>**

**34. Midpoint Formula:**

- If you are given  $(X_1, Y_1)$ ;  $(X_2, Y_2)$  as the endpoints of any line segment, the midpoint,  $(X_m, Y_m)$  is  $\left(\frac{X_1 + X_2}{2}, \frac{Y_1 + Y_2}{2}\right)$

**35. Distance Formula:**

- If you are given  $(X_1, Y_1)$ ;  $(X_2, Y_2)$  as the endpoints of any line segment, the distance is given as  $\sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$

**36. Slope Formula:**

- If you are given  $(X_1, Y_1)$ ;  $(X_2, Y_2)$ , the slope is given as  $\frac{Y_2 - Y_1}{X_2 - X_1}$ :
  - **Parallel lines** have equal slopes
  - **Perpendicular lines** have negative reciprocal slopes
  - **Reflected lines** have negative slopes
  - **Lines reflected in  $y = x$**  have reciprocal slopes

**37. The Equation of a Circle:**

- $x^2 + y^2 = r^2$  represents a circle of radius  $r$  centered at the origin
- $(x - h)^2 + (y - k)^2 = r^2$  represents a circle of radius  $r$  centered at a point  $(h, k)$

**38. The Equation of a Parabola:**

- $y = a(x - h)^2 + k$  represents a parabola with vertex  $(h, k)$ :
  - If  $a > 0$ , the parabola is upward turning and the vertex is a minimum
  - If  $a < 0$ , the parabola is downward turning and the vertex is a maximum

**39. The Equation of a Line:**

- **Slope – Intercept:**  $y = mx + b$ , where  $m$  is the slope and  $b$  is the y-intercept
- **Point Slope:**  $y - y_1 = m(x - x_1)$ , again  $m$  is the slope and  $(x_1, y_1)$  is a point on the line
- **General Form:**  $Ax + By = C$ :
  - Parallel line:  $Ax + By = D$  ( $a$  value other than  $C$ )
  - Perpendicular line:  $Bx - Ay = P$  (any value including  $C$ )

**40. The Equation of an Ellipse:**

- **An ellipse centered at  $(h, k)$  is given by the equation:**  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
- **$2a$  is the major axis (the diameter parallel to the x-axis)**
- **$2b$  is the minor axis (the diameter parallel to the y-axis)**

**ALGEBRA II (about 15% of the exam)****41. Quadratic Analysis ( $ax^2 + bx + c = 0$ ):**

- The discriminant of a quadratic tells you the nature of the roots
  - $D = b^2 - 4ac$ 
    - If  $D < 0$  the roots are imaginary (complex conjugates)
    - If  $D = 0$ , there are two equal roots (double root)
    - If  $D > 0$ , the roots are real and unequal:
      - If  $D$  is a perfect square, the roots are rational
      - If  $D$  is not a perfect square, the roots are irrational
- The sum of the roots is  $-b/a$
- The product of the roots is  $c/a$
- The axis of symmetry is given by  $x = -b / 2a$
- You will not need to know how to use the Quadratic Formula
- To factor  $ax^2 + bx + c = 0$ , find two numbers that multiply to  $ac$ , and add to  $b$ :
  - Example:  $2x^2 - x - 45 = 0$  ( $ac = -90$ ,  $b = -1$ : the two numbers are  $-10$  and  $9$ )
    - Rewrite  $2x^2 - 10x + 9x - 45 = 0$  and factor by grouping:
    - $2x(x - 5) + 9(x - 5) = 0$  which leads to  $(2x + 9)(x - 5)$ .

**42. Complex (Imaginary) Numbers:**

- $\sqrt{-1} = i$ ,  $-1 = i^2$ ,  $-i = i^3$
- To simplify any power of  $i$ , reduce the power to the largest multiple of 4 (which always equals 1), and the remainder of the power:
  - $i^{95} = i^{92} * i^3 = 1 * i^3 = -i$

**43. Absolute Value:**

- To solve an absolute value equation  $|ax + b| = c$ , set up two equations. Remember, though: **if  $c$  is less than zero, the equation has no solutions.**
  - $ax + b = c$  and  $ax + b = -c$
- To solve an absolute value inequality in the form  $|ax + b| > c$ , set up two equations:
  - $ax + b > c$  OR  $ax + b < -c$
- To solve an absolute value inequality in the form  $|ax + b| < c$ , set up one equation:
  - $-c < ax + b < c$  and solve for  $x$

**44. Direct and Indirect Proportions:**

- In a direct relationship, as one variable changes, the other changes at the same rate:
  - $\frac{X_1}{Y_1} = \frac{X_2}{Y_2}$  or  $y = kx$
- In an inverse (or indirect) relationship, as one variable increases, the other decreases at the same rate (and vice versa):
  - $X_1Y_1 = X_2Y_2$  or  $y = \frac{k}{x}$

## 50 Important Math Concepts You Must Know for the ACT

### 45. Matrix Mathematics

- A matrix is a **rectangular** arrangement of **numbers** arranged in **rows** and **columns**. The dimensions of a matrix are shown as  $r \times c$ , namely, the number of (horizontal) rows and then the number of (vertical) columns.
- To add or subtract two or more matrices, each matrix must have the same dimension, and the resulting matrix will have the same dimension.
- To multiply two matrices, whose respective dimensions are  $r_1 \times c_1$  and  $r_2 \times c_2$ ,  $c_1$  must equal  $r_2$ . The resulting matrix will have dimensions of  $r_1 \times c_2$ .
- The determinant of a matrix is a single value that can be determined from a matrix whose rows and columns are exactly the same:

- The ACT only asks students to find the determinant of a matrix of dimensions  $2 \times 2$ .

- If a matrix is given as  $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ , then the determinant of the matrix is calculated by the formula  **$ad - bc$** .

### 46. Logarithms

- Logarithms are the family of inverse functions to exponential functions
- A logarithmic expression returns an exponent, e.g.  $\log_3 9 = 2$
- If  $D^N = A$  in exponential form, then  $\log_D A = N$  is the logarithmic form
- Rules of logarithms:
  - The product of logs is its sum:  $\log(ab) = \log a + \log b$
  - The quotient of logs is its difference:  $\log(a/b) = \log a - \log b$
  - To simplify the power of a log, put the power to the front of the logarithmic expression:  $\log a^n = n \log a$
  - For any base  $c$ ,  $\log_c 1 = 0$



**TRIGONOMETRY (about 5% of the exam)**

*Even if you have not yet taken Algebra 2 and Trigonometry, you can usually get two of the four trig questions knowing these concepts. For any more advanced trig concepts, you will be given the appropriate formula. You can usually use your calculator to answer virtually any trig question.*

**47. Basic Right Angle Trigonometric Relationships**

$$\bullet \text{ Sine} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{Cosecant} = \frac{1}{\text{Sine}}$$

$$\bullet \text{ Cosine} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{Secant} = \frac{1}{\text{Cosine}}$$

$$\bullet \text{ Tangent} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\text{Cotangent} = \frac{1}{\text{Tangent}}$$

**48. Pythagorean Relationships**

- $\sin^2 x + \cos^2 x = 1$
- $\tan^2 x + 1 = \sec^2 x$
- $1 + \cot^2 x = \csc^2 x$

**49. Quotient Relationships**

$$\bullet \text{ Tangent} = \frac{\text{Sine}}{\text{Cosine}}$$

$$\bullet \text{ Cotangent} = \frac{\text{Cosine}}{\text{Sine}}$$

**50. Quadrants where trig values are positive (think of ASTC, “All Students Take Calculus”):**

- Quadrant I: all trig functions
- Quadrant II: Sine and Cosecant
- Quadrant III: Tangent and Cotangent
- Quadrant IV: Cosine and Secant